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THE POSSIBILITY OF PASSING TO A ONE-DIMENSIONAL HYDRODYNAMIC MODEL IN THE PROBLEM OF CONTROL OF THE MIRROR OF A MAGNETOELECTRIC LASER SCANNER WITH HYDRAULIC DAMPING

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A substantiation of replacement of a two-dimensional hydrodynamic model of the flow of a damping liquid in a galvanometer cavity by a one-dimensional model in the problem of control of a galvanometric laser scanner is proposed.

Magnetoelectric scanners with hydraulic damping developed on the basis of high-frequency galvanometers are very promising devices for laser beam deflection. These scanners have a number of technical and economic advantages over widely used magnetodynamic systems, mainly due to a wide range of operating frequencies, low control currents, and technological effectiveness [1, 2].

Most modern deflecting systems of this type do not have a mirror position sensor. This circumstance precludes negative feedback between the control unit and the galvanometer mirror of the scanner. As a result, the working frequency range within which the nonlinearity of the amplitude-frequency response is rather low and a linear conversion of current–mirror rotation angle is provided is limited from above by a frequency lower than the resonance one [3].

The use of electronic filters for nonlinearity compensation in amplitude-frequency and phase-frequency responses has its own disadvantages: the necessity of fine tuning over the entire frequency range and the corresponding complexity of readjustment of the control unit during functioning of the ray deflection system due to changes in operating conditions, aging of the damping liquid, and possible replacements of the galvanometer. Due to the fact that laser beam control assumes the presence of a microprocessor or computer in the control system of the scanner, the variant in which the microprocessor (computer) itself forms the control signal with account for the actual dynamic response of the galvanometer can be considered to be optimum [4].

It is the dynamic response that is the most important and poorly understood component in the general optimization problem, which is also complicated by possible manifestations of rheological properties of the damping liquid [1]. Understanding the basic physical processes on the basis of adequate numerical simulation of the flow structure of the damping liquid in the galvanometer cavity provides the possibility of accounting properly for the dynamic characteristics of the system.

Development of a mathematical model of the dynamics of the movable system of the galvanometer that would adequately describe the behavior (rotation) of the mirror as a function of the control current through the galvanometer frame at each instant of time is very important. However, development of a model of this type is connected with serious problems in the interpretation of the second term in the equation for vibrations of the movable system of the galvanometer [3]

$$I_z\ddot{\varphi} + M_{\text{liq}} + c_w\varphi = M_z\,.$$

Indeed, finding an exact value of M_{liq} at each instant of time is connected with the necessity of solving a system of Navier–Stokes equations for a time-dependent process, which by itself is an involved problem.

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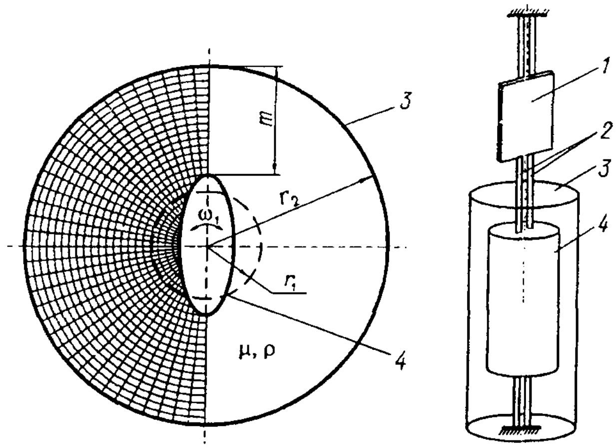

Fig. 1. Cross section of a mounting galvanometer at the level of its frame with a difference grid imposed and schematic of the movable system of the galvanometer: 1) mirror, 2) tension wires, 3) galvanometer casing, 4) galvanometer frame.

Investigations have shown that introduction of a simplified representation for M_{liq} frequently used in solving applied problems [3]

$$M_{liq} = b\dot{\varphi}$$

does not allow for an accurate description of the galvanometer dynamics within the range of high operating frequencies. This is connected with the complex behavior of the damping liquid in the galvanometer cavity and the presence of inertial effects in the liquid that manifest themselves most strongly upon sharp changes in the angular velocity of the frame.

In developing a hydrodynamic model of the flow of the damping liquid in the galvanometer cavity during frame rotation, the following factors should be accounted for. Inasmuch as the height of the galvanometer frame, as a rule, exceeds substantially its mean diameter, we assume that the parameters of the liquid remain constant along the frame axis, i.e., the liquid layers undergo plane motion. Edge effects at the ends of the frame are not taken into account. In this case the model is two-dimensional, and finding a solution based on this model is a complicated and resource-consuming problem. In this case one has to solve a system of partial differential equations in each time step of the control.

We simplify the problem by representing the hydrodynamic model in a one-dimensional formulation. However, a discrepancy exists that is connected with the geometry of the frame cross section. A one-dimensional model can be applied correctly only to frames having a cylindrical shape and correspondingly a circular cross section. The vast majority of frames have a rectangular or elliptical shape of the cross section (see Fig. 1).

Let us assume that a form factor exists for frames with a noncircular cross section that makes it possible to find a correspondence between a noncircular frame and a circular one with a certain diameter that behaves like the corresponding noncircular frame from a dynamics point of view. The effective diameter D_{eff} of the corresponding circular frame coinciding with its actual diameter can play the role of the form factor.

Figure 1 presents a schematic of the movable system and a cross section of a scanner galvanometer based on an M043-type commercial mounting galvanometer, at the level of its frame, with a difference grid imposed.

To approximate derivatives in the Navier–Stokes equations, central differences on a distributed grid employed in the "marker and cell" method [5] were used. This made it possible to relate values of p, V_r, and V_φ at neighboring points and to avoid oscillations in the solution for the peressure.

To find a solution, we used an iterative method of solving the equations of motion of a viscous liquid based on approximating the equations of motion and continuity using an implicit two-level scheme over time and solving the resulting system of nonlinear algebraic equations by an iterative method [5].

We modeled the liquid flow in the galvanometer cavity upon instantaneous setting of the frame in motion. Here the angular velocity of the frame was constant beginning with the instant of actuation. Three integral

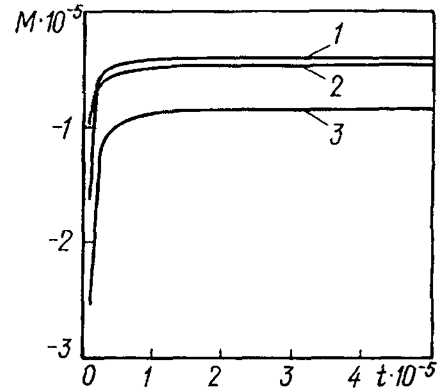

Fig. 2. Time dependence of the total moment of theviscous friction M_{Σ} and its components M_{τ} and M_{σ} in the course of the transient process: 1) M_{τ}, 2) M_{σ}, 3) M_{Σ}. M, N·m; t, sec.

parameters were determined in each time step: the moment due to the tangential stress M_{τ}, the moment due to the normal stress M_{σ}, induced by the noncircular cross section of the frame, and the total moment M_{Σ}, having the meaning of the moment due to the viscous friction. In the case of circular frames the value of the total moment coincides with the value of the moment due to the tangential stress.

An investigation confirmed the existence of a form factor in the form of an effective diameter D_{eff} for frames with various shapes of the cross section [6], including asymmetric ones with a center of gravity shifted from the rotation axis, and the algorithm for its evaluation is as follows. Based on results of solving the two-dimensional problem, time dependences of the total moment (the moment due to the viscous friction) and its components are constructed (see Fig. 2).

The asymptotic behavior of the moments in time bears witness to a change in the character of the motion of the liquid in the galvanometer from a nonstationary to a stationary mode. It should be noted that, due to the presence of this transient process in the liquid, an assumption of the type $M_{\text{liq}} = b\dot{\varphi}$ yields an erroneous result, especially at high vibration frequencies of the galvanometer frame. Indeed, $\dot{\varphi} = \text{const}$, and therefore, the coefficient b cannot be constant.

An analytical solution of the problem of stationary liquid flow between two coaxial cylinders rotating at different but constant angular velocities makes it possible to find an expression for the moment of the viscous friction [7]:

$$M_{\text{cyl}} = \frac{4\pi\mu H_{\text{cyl}} (\omega_2 - \omega_1)\, r_2^2 r_1^2}{r_2^2 - r_1^2} .$$

Taking into account the zero angular velocity of the outer cylinder (the galvanometer casing), the effective diameter D_{eff} can be determined as follows:

$$D_{\text{eff}} = 2 \left(\frac{M_{\text{cyl}}\, r_2^2}{M_{\text{cyl}} - 4\pi\mu H_{\text{cyl}}\, r_2^2 \omega_1} \right)^{1/2} .$$

In this case the moment of the viscous friction M_{cyl} should be understood as the value of the total moment M_{Σ} at $t \to \infty$. By changing the geometry of the original frame to a circular one with the corresponding diameter D_{eff} and

* Here and below, the values of the moments of the friction are negative, since the moments being considered are due to the friction of the liquid acting on the frame and are opposite to the direction of frame rotation. To illustrate, we consider a frame having an ellipsoidal shape of the cross section with a magnitude of the narrow dimension of 180 μm, a width of 380 μm, and height of 6.5 mm, which is placed within a galvanometer casing with the inner Diameter of 1000 μm, and rotates with the angular velocity of 2200 rad/sec in the liquid with the parameters μ = 1.48 Pa·sec and ρ = 953 kg/m^3.

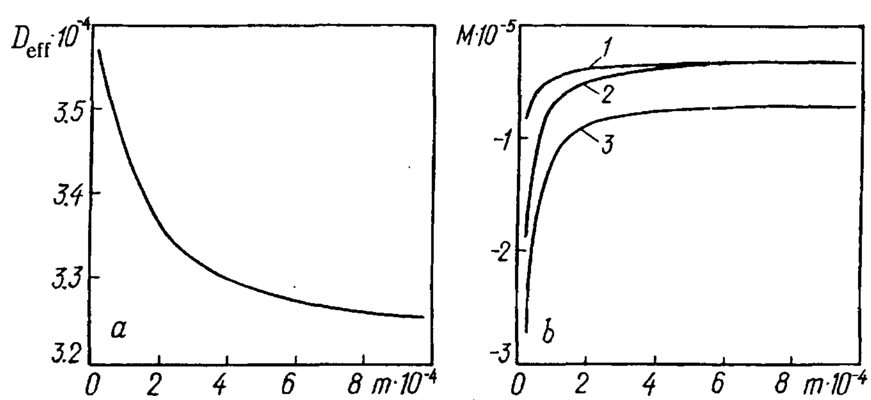

Fig. 3. Effect of the gap size m on the effective diameter D_{eff} with the frame geometry kept constant (a) and on the absolute values of the moments M_τ, M_σ, and M_Σ in the steady process (b): 1) M_τ, 2) M_σ, 3) M_Σ. D_{eff}, m, m.

solving once more the problem of the motion of the liquid in the galvanometer cavity upon instantaneous setting of the frame in motion, one can obtain a similar time dependence of the moment of the viscous friction but already for the circular frame, all other parameters being the same. Coincidence of the moments of the viscous friction in each time step for the circular and noncircular frames within the entire range of working frequencies of the galvanometer substantiates the existence of a form factor.

Let us evaluate the range of applicability of the approach as a whole. We take the Reynolds number Re as a criterion. Investigations showed that when Re ≤ 5 the error in the moment of the viscous friction upon replacing a noncircular frame by a circular one with the corresponding effective diameter does not exceed 1%. It should be noted for comparison that within the entire working frequency range of a scanner based on a commercial galvanometer, angular velocities do not exceed 2200 rad/sec, which, with the geometry and the parameters of the damping liquid taken into account, corresponds to values of the Reynolds number smaller than 0.05. This makes it possible to apply the one-dimensional hydrodynamic model of frame motion to the description of the behavior of most scanners based on commercial galvanometers.

However, the question arises as to whether the effective diameter D_{eff} is a property of the system as a whole (the combination of the frame shape, the inner diameter of the galvanometer casing at the level of the frame, and the parameters of the liquid), or a property of the frame–casing system, or even a property of the frame itself.

A change in the parameters of the liquid such as its density and viscosity affects the value of the moment of the viscous friction and its behavior in time but does not affect the value of the effective diameter D_{eff} (Re ≤ 5). Thus, we conclude that D_{eff} is a property of the geometric parameters of the galvanometer. To determine whether the effective diameter of the frame is a property of the system of frame–galvanometer casing or a property of the frame itself, we carried out the following numerical experiment. We varied the value of the inner diameter of the galvanometer casing at the level of the frame. A change in the inner diameter while the geometric parameters of the frame are kept the same means a change in the quantity m. For example, for the galvanometer used, the quantity m at the narrowest place equals 310 μm for an inner diameter of the casing equal to 1000 μm, and in this case $D_{eff} = 332$ μm. A change in the magnitude of the gap within the limits of 25–980 μm led to a decrease in D_{eff} from 356 to 325 μm, respectively, i.e., by less than 10% (see Fig. 3a).

This result bears witness to the fact that, although the form factor taken in the form of the effective frame diameter is a property of the frame–casing system, in systems in which the value of the inner diameter of the casing substantially exceeds the widest part of the frame the effective diameter is a property of the frame itself, which stems from the asymptotic behavior of D_{eff} as a function of m. This conclusion is rather obvious, since in the limiting case where the inner diameter of the casing tends to infinity, this problem transforms into the problem of a frame rotating in an infinitely extended liquid. The effect of the quantity m on the absolute values of the moments M_τ, M_σ, and M_Σ in the steady process is shown in Fig. 3b. The asymptotic behavior of the moments characterizes the gradual transformation of the problem of a frame rotating in a volume bounded by the galvanometer casing into the problem of a frame rotating in an infinitely extended liquid. Figure 4 demonstrates

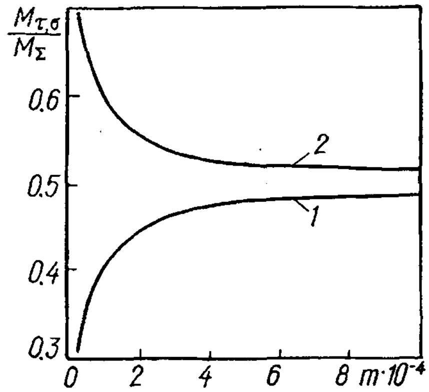

Fig. 4. Contributions of the moments due to the tangential and normal stress M_τ and M_σ to the total moment of the viscous friction M_Σ as a function of the quantity m: 1) M_τ, 2) M_σ.

the effect of the gap on the relative contributions of the moment due to the tangential stress M_τ and the moment of the normal stress M_σ to the total moment of the viscous friction M_Σ. A decrease in the quantity m is characterized by a substantial contribution of the moment of the normal stress M_σ to the total moment of the viscous friction M_Σ.

Passage to the one-dimensional hydrodynamic model and a frame with a circular cross section with the corresponding effective diameter D_{eff} makes it possible to simplify substantially calculations of M_{liq} in the equation for vibrations of the movable system of the galvanometer. Here it becomes possible to carry out adequate modeling of the control of the mirror of a laser scanner in real time by using implicit schemes and applying the sweep method for solving the system of finite-difference equations of liquid dynamics in the galvanometer cavity.

NOTATION

I_z, moment of inertia of the movable system of the galvanometer; M_{liq}, moment due to the friction of the liquid acting on the frame; c_w, rigidity coefficient of the tension wires; M_z, external moment applied to the frame; φ and $\ddot{\varphi}$, angle of frame rotation and its second derivative, respectively; $\dot{\varphi}$, angular velocity of the frame; b, friction coefficient of the frame in the liquid (relaxation coefficient); D_{eff}, effective diameter of the frame; p, pressure; V_r, V_φ, velocity components; M_τ, moment due to the tangential friction stress; M_σ, moment due to the normal friction stress; M_Σ, total moment of the liquid friction; M_{cyl}, moment of the viscous friction in the problem of stationary liquid flow between two coaxial cylinders; μ, viscosity of the liquid; m, gap between the frame and the inner surface of the galvanometer casing; H_{cyl}, cylinder height; r_1 and r_2, radii of the inner and outer cylinders; ω_1 and ω_2, angular velocities of the inner and outer cylinders, respectively; t, time; Re, Reynolds number.

REFERENCES

1. G. L. Roshkovan and P. A. Samosudov, High-Frequency Oscillographic Devices [in Russian], Moscow (1982).
2. G. F. Marshall, "Scanner refinements inspire new users," Laser Focus World, June (1994).
3. Yu. K. Rebrin, Spatial Control of Laser Beams [in Russian], Moscow (1977).
4. A. S. Boreisho and A. F. Leonov, "Laser beam control with wideband magnetic-electric scanners: Technology and application," in: International Symposium on Electronic Imaging Device Engineering, Munich, 21-25 June 1993.
5. R. Peire and T. D. Taylor, Computational Methods in Fluid Mechanics [Russian translation], Leningrad (1986).
6. I. V. Uskov, in: Proceedings of the XVIIth All-Russia Seminar on Gas and Plasma Flows in Nozzles, Jets, and Wakes, St. Petersburg (1997).
7. H. Schlichting, Theory of the Boundary Layer [Russian translation], Moscow (1962).